

mathematical giftedness

a creative scenario

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Identification and development of giftedness is a major task of mathematics teachers worldwide. An early identification of gifted children in mathematics can have a number of benefits, like, providing opportunities for the nourishment of their talent, saving them from burnout, and proper utilisation of mathematical talent in future. Moreover, Johny (2008) argues that the identification and nurturing of mathematical talents is basic to the progress of any nation.

The mathematical giftedness may be conceptualised in different ways (Wieczerkowski, Cropley & Prado, 2000). Heller (2004) equated giftedness with talent by analysing recent literature on giftedness and talent. However, creativity is the premier thinking skill that has been associated with giftedness or talent (Davis, 1991; Gallagher, 1991; Krutetskii, 1976; Renzulli, 1978, 1998; Kattou, Kontoyianni, Pitta-Pantazi, & Christou, 2012). Runco (2005) stated that creative potential is one of the most critical commonalities among the various domains of giftedness. Sriraman (2005) on the basis of Usiskin's (2000) eight-tiered hierarchy of mathematical talent suggested that in the professional realm, mathematical creativity implies mathematical giftedness, but the reverse is not necessarily true. Mann (2006) argued that for the development of the mathematical talent, creativity is essential. Moreover, it (creativity) is the key ingredient of giftedness in all the major models of giftedness—the Three-Ring Conception of Giftedness (Renzulli, 1978, 2005); the WICS Model of Giftedness (Sternberg, 2003); the Munich Model of Giftedness (Heller, Perleth, and Lim, 2005); and the Conceptual Model (Sriraman, 2005).

In the Renzulli's three-ringed conception, giftedness is the interaction among the three clusters:

- (a) above average, although not necessarily superior ability,
- (b) creativity, and
- (c) task commitment.

Likewise, Sternberg (2005) mentioned that WICS is an acronym standing for Wisdom, Intelligence, Creativity, Synthesised, and without a synthesis of these three attributes, someone can be a decent contributor to society, and perhaps even a good one, but never a great one. Furthermore, in the Munich Model of Giftedness (MMG), giftedness is conceptualised as a multifactorised ability construct within a network of non-cognitive (e.g., motivation, interests, self-concept, control expectations) and social moderators which are related to the giftedness factors (predictors) and the exceptional performance areas (criterion variables). Mathematical creativity is one of the giftedness factors (predictors) identified in the MMG.

However, it is the conceptual model that specifically deals with the domain of mathematics. The conceptual model suggests the dynamic relationship between mathematical creativity and mathematical giftedness and asks for paying increased attention to maximising the creative potential of the mathematically gifted students in the ideal classroom. Although, Lekin (2011) remarked that systematic research should be performed to understand the relationship between mathematical creativity and mathematical giftedness.

As creativity plays an important role in the development of gifted behavior, it could be erroneous to exclude creativity from gifted education programs. Mann (2006) mentioned, if mathematical talent is to be discovered and developed, changes in classroom practices and curricular materials are necessary. These changes will only be effective if creativity in mathematics is allowed to be part of the educational experience. Moreover, Kim, Cho and Ahn (2003) and Feldhusen (2005) argued that the identification process needs to move beyond the traditional tests, as they do not identify or measure creativity.

Researchers like Prouse (1964, 1967), Balka (1974), Haylock (1987a, 1987b, 1997) and Kapur (1990), suggested some criteria and frameworks for assessing the mathematical creativity of individuals. The criteria are:

A. Four criteria of Prouse (1964, 1967)

1. The ability to make up or see problems in data or in situations which arouse no particular curiosity in other children.
2. The ability to generate particular results either by finding a common thread of induction or by seeing similar patterns by analogy.
3. The ability to vividly imagine the way things appear in space.
4. The ability to offer more than one acceptable solution to a problem with a solution uncommon or clever.

B. Six criteria of Balka (1974)

1. The ability to formulate mathematical hypotheses concerning cause and effect in a mathematical situation (divergent).
2. The ability to determine patterns in mathematical situations (convergent).
3. The ability to break established mind sets to obtain solutions in a mathematical situation (convergent).
4. The ability to consider and evaluate unusual mathematical ideas, to think through their consequences for a mathematical situation (divergent).
5. The ability to sense what is missing from a given mathematical situation and to ask questions that will enable one to fill in the missing mathematical information (divergent).
6. The ability to split general mathematical problems into specific subproblems (divergent).

C. Ten criteria of Kapur (1990)

1. The ability to recognise patterns in numbers and space.
2. The ability to generalise from particular cases.
3. The ability to see that all intuitive generalisation may not be true.
4. The ability to draw a large number of essentially different conclusions from given hypotheses.
5. The ability to see that some facts can be deduced from other facts or the dependence of facts on one another.
6. The ability to insist on precise definitions and formulations of problems.

7. The ability to make mathematical models.
8. The ability to symbolise.
9. The ability to ask mathematical questions.
10. The ability to recognise the possibility of a large number of answers to a question.

D. Framework of Haylock (1987a, 1987b, 1997)

1. The ability to break from mental sets and overcome fixations.
 - a. The fixation in mathematics may be by the continued use of an initially successful algorithm, even when this becomes inappropriate or less than optimal.
 - b. The fixation may be some sort of self-restriction related to the content-universe of the problem.
2. To use divergent production tests in the assessment of mathematical creativity. The approaches in divergent production tests construction can be:
3.
 - a. Problem solving, where the pupil is given a problem that has many different solutions.
 - b. Problem posing, where the pupil is given a mathematical situation and invited to make up as many and varied problems as possible that can be answered from the given information.
 - c. Redefinition, where the pupil is required repeatedly to re-define the elements of a situation in terms of their mathematical attributes.

The factors (common characteristics) which emerged from the analysis of these criteria and other relevant literature (Hadamard, 1945; Krutetskii, 1976; Gallagher, 1985, 1991; Ernest, 1991; Silver, 1994; Sup, Dong-jou, & Jin, 2003; Mann, 2006; Legin & Lev, 2007; Legin, 2009; Bolden, Harries & Newton, 2010) are:

- (a) the ability to overcome fixations in mathematical situations,
- (b) the ability to formulate mathematical problems, and
- (c) ability to solve a mathematical problem with multiple solutions.

These characteristics are shared by diverse educationists and may be helpful in identifying as well as promoting gifted behavior in mathematics.

A classroom intervention for gifted students in mathematics should possibly include certain exercises for these aspects. This is discussed briefly in the following.

Overcoming fixation in mathematical situations

Fixation is a kind of retroactive phenomenon. We usually try to solve problems on the basis of our prior experiences. But, sometimes prior experiences and methods are not useful in solving problems. The alternative methods are usually more simple, effective and better. Literature in psychology considers functional fixedness (the tendency to think of using objects only as they have been used in the past) and mental set (the impact of past experience on present problem solving; specifically the tendency to retain methods that were successful in the past even if better alternatives now exist) as the factors that interfere with effective problem solving (Baron, 2008). Moreover, fixation in mathematics as pointed out by Haylock (1997) may be the continued use of an initially successful algorithm, even when

this becomes inappropriate or less than optimal or due to some sort of self-restriction related to the content-universe of the problem.

A teacher of mathematics should try to provide certain exercises to test for fixation in mathematics. For example, consider the following problem.

Problem 1: Following arrangement is a way to make four identical compartments by using 9 sticks.



Now, try to make four identical compartments with the help of 8 sticks. (Sharma, 2009)

In this problem, the majority of students may have a fixation in the mind that compartments can be rectangular in shape only, similar to the example given. Only a few creative ones may be able to overcome fixation and obtain the solution shown in Figure 1. Students while solving this problem indulge in trial and error. This involves shift of tactics, a kind of flexibility called adaptive flexibility. Haylock (1997) remarked that the opposite of flexibility is rigidity in thinking. Moreover, Guilford (1967) mentioned that if one is too firmly set on one approach, he is handicapped in doing these problems.

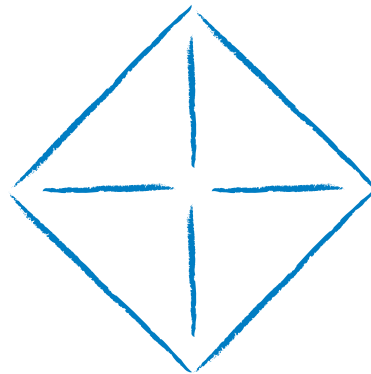


Figure 1. The solution of Problem 1.

Another problem that may provide an experience of overcoming fixation in mathematical situations is:

Problem 2: Suppose you have a barrel of water, a seven cup can, and an eight-cup can. The cans have no marking on them to indicate a smaller number of cups such as 3 cups. How can you measure nine cups of water using only the seven-cup can and the eight-cup can. (Balka, 1974)

The students can respond in various ways to this problem. They may have set in their mind that there is no nine-cup can. Some students may overcome this fixation and indicate that they have been successful in obtaining exactly nine cups, but they may have poured the water on the ground, put the water back in the barrel, or have no container at all to hold the water, thus violating the condition of the task. The creative student will realise the need to keep the water during the process of obtaining nine cups (Balka, 1974; Chauhan, 1977). Another three examples, with regard to the criterion of overcoming fixation are:

Problem 3: Pupils can be given following series of question one by one;

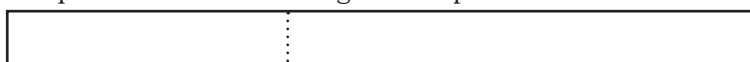
- Find two numbers with sum 10 and difference 4.
- Find two numbers with sum 10 and difference 10.
- Now, two numbers with sum 9 and difference 2. (Haylock, 1997)

Problem 4: Complete the squares so that in each case the rows, columns and diagonals add up to nine. (Haylock, 1987a)

	3			2			2			6	
1	3	5	4	3	2	0	3	6	1	3	5
4											

Problem 5: Cut test.

1 line is required to cut this rectangle into 2 parts:



How many lines are necessary to cut it into:

3 parts?



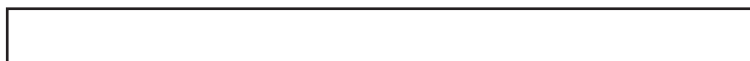
5 parts?



7 parts?



9 parts?



(Haylock, 1987a)

Proposing mathematical problems

The *Curriculum Evaluation and Standards for School Mathematics* (NCTM, 1989) recommended that students should have some experience recognising and formulating their own problems, an activity that is at the heart of doing mathematics. Likewise, NCF (2005) indicated that school mathematics curriculum should include some problem posing exercises. Moreover, Jay and Perkins (1997) claimed that the act of finding and formulating a problem is a key aspect of creative thinking and creative performance in many fields, an act that is distinct from and perhaps more important than problem solving.

Problem posing has three distinct forms, namely, pre-solution posing, in which one generates original problems from a presented stimulus situation; within-solution posing, in which one reformulates a problem as it is being solved; and post-solution posing, in which one modifies the goals or conditions of an already solved problem to generate new problems (Silver, 1994). The author feels that students normally get adequate experience of within-solution posing and to some extent of post-solution posing, but they rarely stretch their mind for pre-solution posing, which is key to mathematical creativity. In problem posing, the students may be asked to generate

mathematical problems from the stimulus provided. The emphasis is on generation of varied and complex problems. The solution of these problems is not primary. The students may enjoy doing this, when they are asked, “Pose such a problem for your friends that they are not able to solve.”

Some examples of the problem posing exercises that may stretch the student’s creative thinking in mathematics are given below:

PP 1: Write three different questions that can be answered from the information below.

Jerome, Elliot, and Arturo took turns driving home from a trip. Arturo drove 80 miles more than Elliot. Elliot drove twice as many miles as Jerome. Jerome drove 50 miles. (Silver & Cai, 1996)

Silver & Cai (1996) reported that on this problem posing task, 509 sixth and seventh grade middle school students provided a total of 1465 responses, out of which more than 70% of the responses were classified as mathematical questions. This indicates the task’s relevance for assessing and encouraging problem posing among mathematics students.

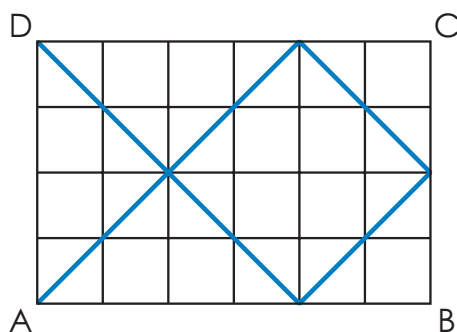
PP 2: Make as many equations as you can so that ‘2’ is at least one of the solutions. (Sharma, 2009)

The PP 2 is used by Sharma and Sansanwal (2012) in a mathematical creativity test for school students. A total of 176 responses is reported in 12 different categories such as linear equation with one variable, quadratic equations, and equation with fraction in constant and/or variable term. Like, solve $x^4 = 16$; $x^5 + 1 = 33$; $\sqrt{x^2} = 2$; $x - y = 0$ [put $y = 2$]; and $a + b = 4$, $a - b = 0$.

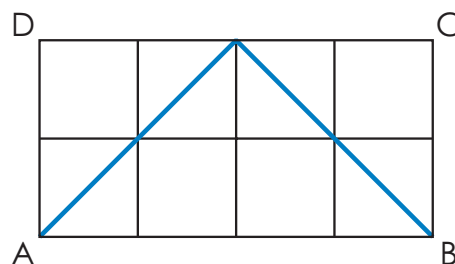
PP 3: Pose a problem. The answer is: “Speed of A = 37 m/min; Speed of B = 28 m/min.” (Leung, 1997)

PP 4:

- (i) Imagine billiard ball tables like the ones shown below. Suppose a ball is shot at a 45° angle from the lower left corner (A) of the table. When the ball hits a side of the table, it bounces off at a 45° angle. In each of the examples shown below, the ball hits the sides several times and then eventually lands in a corner pocket. In Example 1, the ball travels on a 6-by-4 table and ends up in the pocket D, after 3 hits on the sides. In Example 2, the ball travels on a 4-by-2 table and ends up in pocket B, after 1 hit on the sides.



Example 1



Example 2

- (ii) Look at the examples, think about the situation for tables of other sizes, and write down any questions or problems that occur to you.
- (iii) As you work out your solution to the problem, other questions may also come to mind. Write down any questions or problems that occur to you. (Silver, Mamona-Downs, Leung & Kenney, 1996)

PP 5: If the answer is 30, what might be the questions? (Sharma, 2009)

These problems permit the posing of interesting mathematical problems. The students may pose questions in the variety of contexts. The questions posed can be evaluated in terms of fluency, flexibility and originality or mathematical solvability and complexity.

Multiple solution mathematical problems (MSMP)

Problem solving is the main issue of mathematics teaching since its origin (Warburton, 1985). Problem solving is a match between the problem and knowledge one already possesses, thereby resulting in possible solution. Thornburg (1973) argues that without the prerequisite concepts and rules, solutions to many problems would not be readily available and may often be completely undiscoverable. The author, while working with school students, observed that students with better previous knowledge of mathematics were able to find more solutions qualitatively and quantitatively. Gallager (1991) mentioned that a strong knowledge base allows more effective automaticity. A large variety of problem solving activities in mathematics are available. The kind of problem solving that is related to mathematical creativity is a divergent one, i.e., finding multiple solutions of a mathematical problem. Some MSMP are presented to demonstrate the type of questions teachers need to frame and ask in the classrooms. A number of varied responses can be generated to each of these problems. The author administered MSMP 2 on 15 pre-service secondary school teachers currently enrolled under him. There was significant variance in the quality of responses made by the pre-service teachers. Some of the responses made by two, out of these 15 pre-service teachers, are given in the Table 1. The MSMP 2 can produce similar results with school students. Also, MSMP 6 has been used effectively for testing (Haylock, 1987b) and fostering (Fetterly, 2010) of mathematical creativity at different levels. Likewise, the other problems can be used for testing and encouraging mathematical creativity.

MSMP 1: Use the symbols $+$, $-$, \times , \div and $()$, if needed, to write as many true equations as possible with the numbers 2, 5, 9 using them in same or different order. (Chauhan, 1984)

MSMP 2: Add, first ten numbers, i.e. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 in as many different ways you may imagine. (Tuli, 1981)

Table 1

Student teacher 1	Student teacher 2
$\begin{aligned}\text{Sum} &= (1+2)+(3+4)+(5+6)+(7+8)+(9+10) \\ &= 3 + 7 + 11 + 15 + 19 \\ &= 10 + 26 + 19 \\ &= 55\end{aligned}$	$\begin{aligned}S_n &= \frac{n}{2} 2a + (n-1)d \\ &= \frac{10}{2} 2 \times 1 + (10-1)1 \\ &= 5(2+9) \\ &= 55\end{aligned}$
$\begin{aligned}\text{Sum} &= (1+2+3)+(4+5+6)+(7+8+9)+10 \\ &= 6 + 15 + 24 + 10 \\ &= 21 + 34 \\ &= 55\end{aligned}$	$\begin{aligned}\sum n &= \frac{N(N+1)}{2} \\ &= \frac{10(10+1)}{2} \\ &= 55\end{aligned}$

MSMP 3: Classify the following numbers in as many different ways as possible. Write the rule of classification in each case:

$$\frac{-3}{2}, 0, \frac{5}{6}, 100, 2, 5, 1.2, 4.6, -37, \frac{100}{36}$$

(Moghe, 1996)

MSMP 4: Draw as many figures as you can so that the perimeter of each figure is 24 m. Try to find the figure of minimum and maximum area. (Sharma, 2009)

MSMP 5: Suppose the chalkboard in your classroom was broken and everyone's paper was thrown away; consequently you and your teacher could not draw any plane geometry figures such as lines, triangles, squares, polygons, or any others. The only object remaining in the room that you can draw on was a large ball or globe used for geography. List all the things which could happen as a result of doing your geometry on the ball. Let your mind go wild in thinking up possible ideas. (Balka, 1974a)

MSMP 6: Find shapes with areas 2 cm^2 which can be formed by joining dots on a nine-dot centimetre square grid. (Haylock, 1987b)

The quality and quantity of responses given by the students may provide appropriate measure of students' creative ability in mathematics and in turn giftedness in mathematics. The solutions can be evaluated in terms of fluency (the number of valid responses produced), flexibility (shift in thinking) and originality (uncommonness or the statistical infrequency of the response). However, Haylock (1997) pointed out that in a mathematical context the criterion of fluency often seems less useful for indicating creative thought than flexibility.

Another dilemma that arises in the testing of mathematical creativity is in connection with the appropriateness of the response. A response may be highly original, but is of no value if it is not correct in terms of mathematical judgment. However, Haylock (1997) argued that conflicts may occur, in deciding how to respond to a pupil who shows imagination and originality with an irritating level of inaccuracy; teaching mathematical routines while encouraging the willingness to break from stereotyped procedures; and between being systematic and being flexible. The author believes that during the specific testing, it would be more pertinent to consider only those

responses as valid that are mathematically correct, however while teaching a teacher may occasionally detour and encourage imagination and originality even if it is inaccurate.

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